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LETTER TO THE EDITOR

Spreading of damage in a 3D Ising spin glass

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Abstract. We study the time evolution of the Hamming distance of two configurations by Monte Carlo simulation, differing only by the central spin, of the $\pm J$ 3D Ising spin glass. We observe two temperature regimes: a high-temperature one where the damage readily spreads and a low-temperature one where the spreading of damage is hindered. The critical temperature numerically coincides with the spin-glass temperature found by Monte Carlo simulation.

Spin glasses are complex physical systems with dynamic and static effects which have been observed experimentally (for a review on spin glasses see [1]). The dynamic aspects (remanent magnetisation, the susceptibility cusp shift with the frequency of the AC field, etc) due to relaxation effects, suggested for many years that spin glasses did not have a true thermodynamic phase, but only a dynamic one. Recently, however, heavy computational effort [2] and phenomenological scaling arguments at zero temperature [3] suggested a true phase transition. We intend here to make a study of the dynamics of the spin glass by analysing the temporal evolution of the Hamming distance between two configurations of the $\pm J$ three-dimensional Ising spin glass and its 'total damage' time. Early works [4-6] studying the propagation of a small damage in magnetic systems (spreading of damage) have shown a strong correlation between the frozen phase and the thermodynamic one. As the freezing temperature obtained here practically coincides with the best known result for the spin glass temperature we also support this correlation.

The spreading of damage was investigated in the 2D Ising model and in the *Q2R* by Stanley *et al* [4] using Glauber dynamics. They found a critical temperature equal to the Curie temperature (T_c). Costa [5] studied the spreading of damage in the 3D Ising model (Glauber dynamics) and found $T^* = 0.96T_c$ (the threshold temperature between the chaotic and frozen phases). He speculates that previous coincidences of these temperatures may have been fortuitous, as one transition is static and the other is dynamic and is not understood [7].

Derrida and Weisbuch [6] (henceforth referred to as DW) investigated spreading of damage in the 3D $\pm J$ Ising spin glass, using heat bath dynamics. Besides the usual small damage spreading they also studied the cases where the spins are randomly orientated on both systems and also the case where the two systems are exactly inverted. They found three temperature regimes: a high-temperature one where the long-times Hamming distance of the three cases is zero, an intermediate-temperature one where

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the Hamming distances are the same and non-zero, and a low-temperature one where the Hamming distances are different. The threshold temperatures are $T_1 = 1.8J$ and $T_2 = 4.1J$.

So far, spreading of damage has been done exclusively through computer experiments. The method is as follows: take a system configuration (A) in equilibrium at a given temperature; make a copy of it (B); introduce a 'damage', by changing the state of one or more of the sites; treat both systems in the same way using the same algorithm and using the same random numbers; study the temporal evolution of the damage. This process is then repeated for a set of temperature values.

Some quantities are used to measure the damage. One of them is the Hamming distance (M), which measures the extent of the damage at a given time. It is defined as the fraction of the sites that are different in the two lattices. Another quantity is total damage time. It is defined as the time required for all the sites to have been damaged at least once. This is a good quantity to distinguish the phase where the damage spreads freely (chaotic) from the one in which it is hindered (frozen).

One interesting point is the radical difference between the results obtained by DW and by Costa. As an example, the high-temperature Hamming distance is zero in DW, while in Costa it is $\frac{1}{2}$.

A detailed study by Stauffer [8] offered an explanation for this divergence. At first sight there is very little difference between the 'heat bath' dynamics and the Glauber dynamics.

In the Glauber dynamics a spin is flipped with probability $\exp(-\beta\Delta E)/(1+\exp(-\beta\Delta E))$, where $\Delta E = 2S_i \Sigma J_{ij}S_j$ is the energy difference between the new and old configurations. Defining $k_B h_i \equiv \Sigma J_{ij}S_j$, the spin is then flipped with probability $p = 1/[1+\exp(2h_iS_i/T)]$ and remains unchanged with probability $1-p = 1/[1+\exp(-h_iS_i/T)]$. Therefore is $S_1 = 1$, then it will flip to -1 with probability $1/[1+\exp(+2h_i/T)]$, and remain 1 with probability $1/[1+\exp(-2h_i/T)]$, while, if $S_1 = -1$, then it will remain -1 with probability $1/[1+\exp(+2h_i/T)]$, and flip to 1 with probability $1/[1+\exp(-2h_i/T)]$.

In the heat bath dynamics the spin becomes 1 with probability $1/[1+\exp(-2h_iS_i/T)]$ and becomes -1 with probability $1/[1+\exp(+2h_iS_i/T)]$, regardless of the original value of the spin. It is easy to see that this is entirely equivalent to the previous case.

The difference, therefore, does not lie in the probabilities, but in how the random numbers are used. To see the difference, let us take a particular example. Let us assume that a given site in the lattice A, $S_i^A = 1$, and in lattice B, $S_i^B = -1$, but $h_i^A = h_i^B$. Now S_i^A will flip with probability $p = 1/[1+\exp(+2h_i/T)]$, while S_i^B will flip with probability $1-p = 1/[1+\exp(-2h_i/T)]$. In both cases a random number z is obtained. In the Glauber dynamics S_i^A flips from 1 to -1 if and only if $z < p$ and S_i^B flips from 1 to -1 if and only if $z < 1-p$. In the heat bath dynamics S_i^A flips from 1 to -1 if and only if $z < p$ and S_i^B flips from 1 to -1 if and only if $z > p$. Therefore, in the case of the Glauber dynamics three possibilities exist: (a) $z < \min(1-p, p)$ both spins flip; (b) $\min(1-p, p) < z < \max(p, 1-p)$ one spin flips; (c) $\max(1-p, p) < z$ no spin flips. On the other hand, in the heat bath case there are only two possibilities: (a) $z < p$ S_i^A flips; (b) $z > p$ S_i^B flips. Therefore one and only one always flips and the result is always that they become equal. This effect is particularly striking at high temperatures when $p = 1-p = \frac{1}{2}$. Here option (b) for Glauber dynamics does not apply so that either both spins flip or both do not, so that they remain different. For the heat bath dynamics one must flip and they must become equal. The conclusion is then that at high temperatures the configurations will become more and more similar with the heat bath dynamics, while they will become, slowly, more and more different with Glauber dynamics.

Apart from the usual one-site damage, other drastically different initial conditions can be considered, such as: (a) reversed lattices ($M=1$); (b) both lattices random ($M=\frac{1}{2}$). While using Glauber dynamics, however, they only produce trivial results. Case (a) will remain $M=1$, for all times, for all temperatures, because the equivalent sites on both lattices will yield the same value of p, and if $z<p$ both flip, otherwise both remain unaltered, so that M continues to be 1. It can also be argued that in case (b) M will remain close to $\frac{1}{2}$. At high temperatures this is beyond question. At low temperatures, if one accepts that the distribution of ground levels in the phase space is reasonably uniform, there should be one not too far from the system starting point and the Hamming distance should be $\frac{1}{2}$, on average. We have actually checked that and M is very close to $\frac{1}{2}$. We see, then, that the large interval of temperatures for which the Hamming distance is independent of the initial conditions found by DW is not repeated here, showing that this feature is dependent on the dynamics used.

We have applied the Glauber dynamics for the $\pm J$ 3D Ising spin glass and, as expected, we obtained results consistent with Costa and different from DW. We started with a fairly small system (10^3 sites). To calculate the Hamming distance, only the surviving samples (i.e., those whose Hamming distance did not become zero) were used. For each temperature we make the configurational average of the temporal evolution of the Hamming distance. This temporal evolution presents a transient stage and after a temperature-dependent time t^* it seems to oscillate only around the mean value. The value of the Hamming distance is then obtained by taking a time average of the values for times later than t^*. Figure 1 shows the Hamming distance plotted

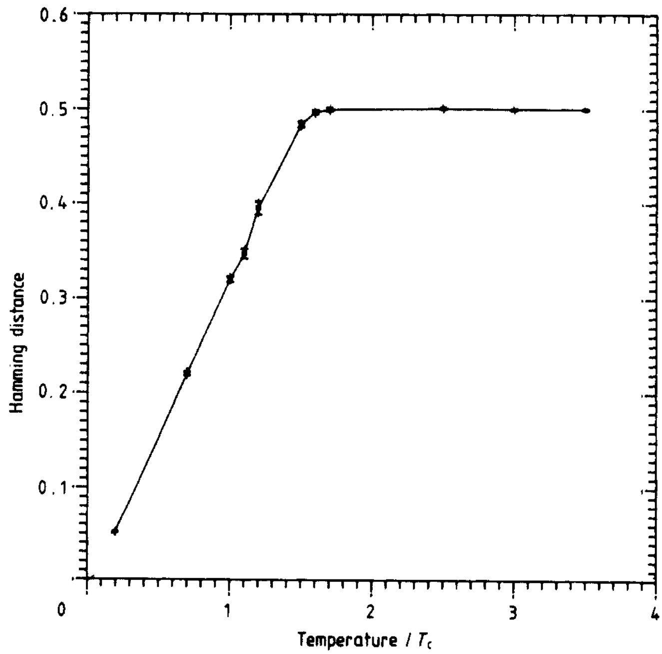

Figure 1. Hamming distance plotted against temperature. At low temperature it decreases and seems to go to zero at $T=0$. At higher temperatures it rises to reach a plateau at the value $\frac{1}{2}$.

against temperature. At low temperature it decreases and seems to go to zero at $T=0$. At higher temperatures it rises to reach a plateau at the value $\frac{1}{2}$. This seems to be physically justifiable, because, since the spins are totally uncorrelated at high temperatures, we should expect that, on average, half of the spins should be different at a given time.

To obtain the mean total damage time we have performed a configurational average, over at least 60 samples, for each temperature. We have also calculated the median of the total damage time and verified that it coincides with the mean practically at all temperatures. Therefore, we used the median for $T \leqslant 1.1 T_c$ (we use $T_c = 1.2J$, from [2]), because there were samples which had not reached the total damage with 20 000 time steps, which was the largest value used. For higher temperatures we used the mean, because it gives error bars. This result is depicted in figure 2. We can see that it goes to infinity both at infinite temperature and at and below the freezing temperature. We reached the conclusion that T^* is close to T_c. We can identify only two regimes here: one above T^*, where the damage can readily spread and another below it, where it is hindered. In fact, this divergence of the total damage seems to be a good indication of a phase transition. The connection between the threshold temperature in spreading of damage with percolation suggested by Costa for the ferromagnetic case, is not clear [7] in our case and deserves further consideration. At infinite temperature, as we have already argued, the flip probability is $\frac{1}{2}$ (regardless of the state of its neighbours), and either the spins on the two lattices flip together or remain as they were together. Therefore the initial damage remains the only one and the whole lattice can never be completely damaged.

Although there is still work to be done, it seems safe to reach the conclusion that there are two and not three phases in the 3D Ising spin glass, using Glauber dynamics.

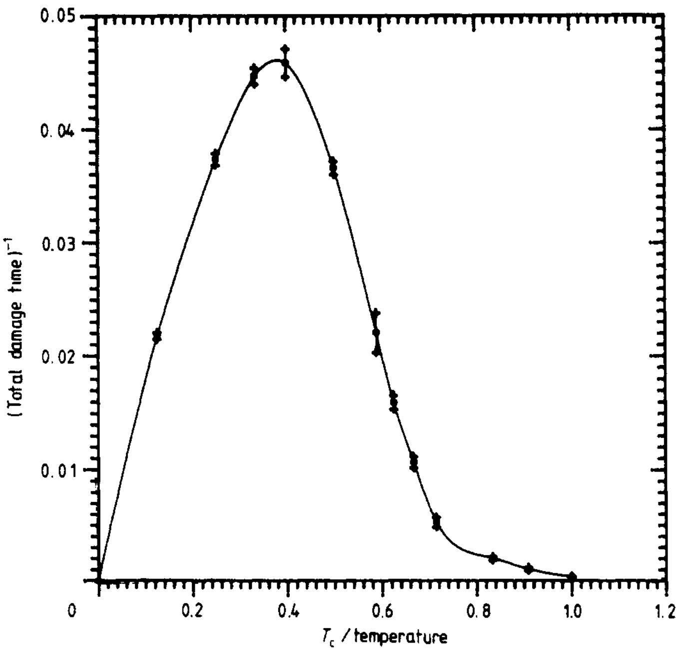

Figure 2. Inverse total damage time plotted against inverse temperature. We can identify two regimes here: one above T^*, where the damage can readily spread, and another below it, where it is hindered.

Also T^*, if not at it, is very close to T_F (critical temperature for the spin glass, obtained by Bhatt and Young and by Ogielsky and Morgenstein [2]). It becomes obvious that the details of the dynamics used are averaged out and give the same results in many cases, but this is certainly not so here.

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